



DISCRETE INVERSE AND STATE ESTIMATION PROBLEMS

With Geophysical Fluid Applications

CARL WUNSCH

*Department of Earth, Atmospheric and Planetary Sciences
Massachusetts Institute of Technology*



CAMBRIDGE
UNIVERSITY PRESS

Contents

| | |
|--|---------|
| <i>Preface</i> | page ix |
| <i>Acknowledgements</i> | xi |
| Part I Fundamental machinery | 1 |
| 1 Introduction | 3 |
| 1.1 Differential equations | 4 |
| 1.2 Partial differential equations | 7 |
| 1.3 More examples | 10 |
| 1.4 Importance of the forward model | 17 |
| 2 Basic machinery | 19 |
| 2.1 Background | 19 |
| 2.2 Matrix and vector algebra | 19 |
| 2.3 Simple statistics: regression | 29 |
| 2.4 Least-squares | 43 |
| 2.5 The singular vector expansion | 69 |
| 2.6 Combined least-squares and adjoints | 118 |
| 2.7 Minimum variance estimation and simultaneous equations | 125 |
| 2.8 Improving recursively | 136 |
| 2.9 Summary | 143 |
| Appendix 1. Maximum likelihood | 145 |
| Appendix 2. Differential operators and Green functions | 146 |
| Appendix 3. Recursive least-squares and Gauss–Markov solutions | 148 |
| 3 Extensions of methods | 152 |
| 3.1 The general eigenvector/eigenvalue problem | 152 |
| 3.2 Sampling | 155 |
| 3.3 Inequality constraints: non-negative least-squares | 164 |
| 3.4 Linear programming | 166 |
| 3.5 Empirical orthogonal functions | 169 |
| 3.6 Kriging and other variants of Gauss–Markov estimation | 170 |

| | | |
|-----|---|-----|
| 3.7 | Non-linear problems | 171 |
| 4 | The time-dependent inverse problem: state estimation | 178 |
| 4.1 | Background | 178 |
| 4.2 | Basic ideas and notation | 180 |
| 4.3 | Estimation | 192 |
| 4.4 | Control and estimation problems | 214 |
| 4.5 | Duality and simplification: the steady-state filter and adjoint | 229 |
| 4.6 | Controllability and observability | 232 |
| 4.7 | Non-linear models | 234 |
| 4.8 | Forward models | 248 |
| 4.9 | A summary | 250 |
| | Appendix. Automatic differentiation and adjoints | 250 |
| 5 | Time-dependent methods – 2 | 256 |
| 5.1 | Monte Carlo/ensemble methods | 256 |
| 5.2 | Numerical engineering: the search for practicality | 260 |
| 5.3 | Uncertainty in Lagrange multiplier method | 269 |
| 5.4 | Non-normal systems | 270 |
| 5.5 | Adaptive problems | 273 |
| | Appendix. Doubling | 274 |
| | Part II Applications | 277 |
| 6 | Applications to steady problems | 279 |
| 6.1 | Steady-state tracer distributions | 280 |
| 6.2 | The steady ocean circulation inverse problem | 282 |
| 6.3 | Property fluxes | 309 |
| 6.4 | Application to real oceanographic problems | 311 |
| 6.5 | Linear programming solutions | 326 |
| 6.6 | The β -spiral and variant methods | 328 |
| 6.7 | Alleged failure of inverse methods | 331 |
| 6.8 | Applications of empirical orthogonal functions (EOFs) (singular vectors) | 333 |
| 6.9 | Non-linear problems | 335 |
| 7 | Applications to time-dependent fluid problems | 340 |
| 7.1 | Time-dependent tracers | 341 |
| 7.2 | Global ocean states by Lagrange multiplier methods | 342 |
| 7.3 | Global ocean states by sequential methods | 351 |
| 7.4 | Miscellaneous approximations and applications | 354 |
| 7.5 | Meteorological applications | 356 |
| | <i>References</i> | 357 |
| | <i>Index</i> | 367 |
| | <i>Colour plates between pp. 182 and 183.</i> | |